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Ethnomathematics Projects for Projective Geometry in Distance Learning

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Synopsis

This article reports on an ethnomathematics project we developed and implemented in an online learning environment to connect with students' cultural backgrounds and to highlight the links between their environment, culture, and mathematics. The project is based on Simple House Theory, a framework that involves the application of projective geometry in the design of traditional houses and building numbers. Our results show that this approach fostered active participation and engagement despite the standard constraints and challenges of distance learning.

Key words: distance learning, projective geometry, simple house theory, building numbers, projective coordinates

1. Introduction

The COVID-19 pandemic has compelled universities worldwide to shift their teaching and learning processes to online environments. However, it is well known that the transition to online learning presents several challenges, particularly in terms of controlling the teaching and learning process and motivating students; see for example [18, 11]. The prevalence of distractions, such as video games and social media, while students interact with computer monitors or smartphones, further adds to the complexity faced by educators [28].

Consequently, promoting self-control among students has become essential in the context of online and distance learning [29].

Recognizing the significance of engaging students in the teaching and learning process and believing that the integration of students' cultural perspectives can enrich their learning experience in virtual settings, we decided to address the challenges of online learning described above by leveraging students' cultural backgrounds. To that end, we designed an ethnomathematics project which we hoped would enable students to grasp mathematical concepts by establishing connections between their environment, background, culture, and mathematics, and thereby encourage their active participation in distance learning [28].

In this paper, we describe our ethnomathematics project and present the outcomes of implementing it in the context of distance learning during the COVID-19 pandemic. By analyzing data collected from students enrolled in a collaborative course between the State University of Makassar (UNM Makassar), Indonesia, and Tunghai University, Taiwan, we examine the impact of leveraging cultural backgrounds on students' motivation and self-control in the online learning environment. Through a quantitative analysis of surveys, tests, and projects, we aim to provide insights into the effectiveness of this innovative approach and its implications for future online teaching and learning practices.

2. Ethnomathematics

Ethnomathematics, which explores the cultural aspects of mathematics, offers a promising approach to foster meaningful engagement and enhance students' self-directed learning [14, 20, 28]. Both a theoretical construct and a practical approach to mathematics and its education, ethnomathematics emerged as a response to the dominance of Western mathematics, which supplanted local mathematical practices and knowledge systems during the era of colonization [1]. It is undeniable that various cultures around the world have long practiced their unique mathematical methods for purposes like house construction, measurement, and trade [10]. As a field of study, therefore, ethnomathematics explores the relationship between mathematics and culture, delving into mathematical concepts embedded within daily activities, traditions, myths, and religious practices.

Ethnomathematics encompasses a diverse range of mathematical concepts employed in different cultural settings [19]. It extends beyond the conventional notions of mathematics, ethnicity, and multiculturalism, integrating philosophy, linguistics, pedagogy, anthropology, and history to comprehend socio-cultural environments [22]. The discipline accentuates the significance of contextual factors, such as environmental influences, reasoning methods, and cultural symbols, in shaping mathematical practices within specific communities.

Ethnomathematics involves diverse perspectives on mathematical knowledge and problem-solving techniques. It reflects the distinctive ways cultures mathematize their challenges to derive practical solutions [22]. For instance, the concepts of geometry find expression in various aspects of traditional life, from building structures to artistic creations. Different cultures may interpret arithmetic in their production and trade practices, leading to unique approaches to counting, ordering, measuring, and classifying [9].

Since its inception in 1976,[1] ethnomathematics has been extensively studied across fundamental mathematical concepts, including algebra, geometry, and number theory, showcasing how diverse societies approach mathematical ideas in fundamentally different ways [22, 26, 31, 33]. For instance, ethnomathematical number theory explores counting systems used by ancient Mayans, African cultures, and Papua New Guineans, which significantly differ from the Western numeric systems. Geometry is deeply embedded in the architecture and design of various cultures. African societies in the Sahara region and Mozambican peasantry employ circular or rectangular bases for traditional houses, constructing them without necessarily following step-by-step right angles [33]. Balinese houses and Borobudur Temple in Central Java, Indonesia, showcase symmetry through isometric transformations like translation, rotation, and reflection [25, 31]. Simple House Theory, the main ethnomathematical framework used in this paper, involves connecting projective geometry to a range of traditional houses, mostly from Indonesia [5].

Ethnomathematics serves as a bridge to unravel the multifaceted nature of mathematical practices across cultures, unveiling the richness of mathematical thought beyond Western frameworks. By appreciating the cultural dimensions of mathematics, educators can foster a more inclusive and diverse

[1] See [10, 21] for a brief overview of the history of ethnomathematics.

approach to mathematics education, offering valuable insights into mathematical knowledge systems from cross-cultural, anthropological, and political perspectives [19]. Embracing the diversity of ethnomathematics enables us to envision the future role of mathematics education in multi-cultural societies, empowering learners to appreciate and apply mathematics within their unique cultural contexts.

3. Students' Engagement in Distance Learning

Distance learning, as defined by Bozkurt [4], is the use of digital technology to facilitate formal, informal, and non-formal learning processes, engaging learners emotionally and physically, and promoting increased interaction among facilitators, learners, and learning materials. Bacich and Trevisani [3] emphasize the role of technology in mediating the resources involved in the distance learning process, enabling students to connect with knowledge, establish flexible study routines, and access personalized learning through learning management systems.

To maintain student engagement in an online learning environment, providing various options is crucial, as highlighted by Conrad and Donaldson [8]. These options include empowering students to set their learning objectives, fostering group collaboration, encouraging investigation of relevant resources to address essential questions, employing transdisciplinary and authentic tasks with real-world applications, implementing continuous and performance-based evaluation, and encouraging students to share their work with a broader audience outside the classroom, thus instilling a sense of value and impact.

The principles of engagement in distance learning align well with constructivism, as proposed by [24]. According to constructivist principles, learning is built upon experiences, and individuals actively interpret information to create meaning dynamically through their experiences. Learning is viewed as a collaborative process involving multiple perspectives, where negotiation of meaning takes place.

Weigel [30] supports the idea that content forms the foundation for knowledge creation, making learning more meaningful. Constructing knowledge is seen as a viable process that fosters coherent knowledge structures, creativity, critical analysis, and skilled performance. There is substantial evidence suggesting that learners excel when actively constructing their knowledge [7]. However, there are instances where learners may benefit from explicit guidance or the provision of critical information to aid their progress.

In summary, distance learning leverages digital technology to facilitate diverse forms of learning and engagement, with constructivism serving as a guiding principle to create meaningful and collaborative learning experiences. By providing students with choices and fostering active knowledge construction, distance learning can empower learners to achieve their learning objectives effectively.

Engagement, as defined by Kuh and Hu [16], refers to the quality of students' efforts in educationally purposeful activities that directly contribute to desired outcomes. In the present study, engagement is understood as active interaction with relevant phenomena that aids learning. Student engagement strongly correlates with academic achievement, serving as a measure of success in the teaching and learning process. Positive engagement is manifested through attentive behaviors and positive attitudes, acting as a deterrent to negative behaviors. Assigning students challenging tasks, such as projects, increases their engagement, as they enthusiastically overcome obstacles and contribute to their academic success [13, 2, 6].

According to Ali and Hassan [2], student engagement encompasses three dimensions: behavior, emotion, and cognition, and it can be categorized into positive engagement, indirect engagement, and negative engagement. Positive engagement involves active participation, attending online classes, and achieving success. Conversely, negative engagement occurs when students feel disinterested and skip classes. In online learning, lecturers strive to foster positive engagement through well-designed activities that prevent unexpected disengagement.

Student evaluation of their engagement in the online learning process helps gauge their level of enjoyment and interest in learning. Pleasure and enthusiasm drive students to prepare for future classes, presentations, or tests. Engaging in the teaching and learning process motivates students to invest greater effort in their performance and demonstrate respect by preparing thoughtfully for presentations. Additionally, challenging learning experiences can be an indicator of students' commitment and active involvement in the class discussions.

Lecturers play a crucial role in encouraging positive engagement in online learning. To achieve this, they should provide a conducive course meeting platform, explain concepts clearly, offer feedback, and create an inclusive learning environment. Students, in turn, should actively develop their intellectual and critical thinking skills, recognizing the relationship between course materials and culture. Engaging in discussions and presentations enables them to improve their understanding while enriching their intercultural knowledge and awareness by collaborating with peers from diverse cultural backgrounds, including students from other countries.

Figure 1 illustrates the Engage Learning framework applied to our work here, highlighting how certain cultural features profoundly influence students' lives within a society. This influence extends to their activities, shaped by their parents and community, and presents a valuable resource for enhancing the teaching and learning process [17, 12, 23].

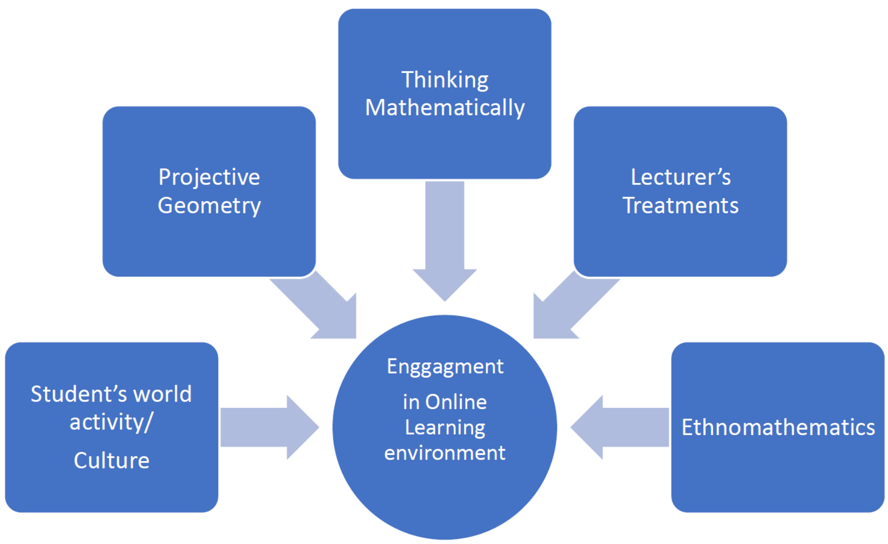


Figure 1: The Engaged Learning framework as applied to the context of this paper.

The teaching and learning process in this framework is designed to leverage students' cultural backgrounds within the ethnomathematics project, incorporating materials on projective geometry and knot theory delivered through distance learning. After comprehending the topology and projective geometry concepts, students are encouraged to think and propose their ideas for their projects. Lecturers provide criteria in the form of questions related to the project, and students complete their tasks and present their projects. This approach fosters creative thinking among students, as they prepare and analyze their ideas, developing critical thinking skills that significantly contribute to their academic achievement. The supportive behaviors of lecturers towards students play an essential role in this process.

4. Simple House Theory

Traditional structures feature distinctive architecture that represents the culture, identity, and history of a country [27]. We have developed Simple House Theory based on an exploration of traditional buildings and projective geometry [5]. Traditional houses are those that have been constructed in a conventional manner. Every traditional home has a concept that guides the design of the structure [32]. In our work we mostly focused on the structure of traditional Indonesian dwellings. These houses have socio-cultural roles and are used by locals from generation to generation.

4.1. Simple House Categorization

Simple dwellings are categorized into three categories according to projective geometry [5]. To get to our categorization, we needed to first represent traditional dwellings via *house pictures*, which were then projected onto a *house diagram* using a number of vanishing points (see Figure 2):

Figure 2: Left: The representation A of a traditional house. Right: The corresponding house diagram $\hat{A}$ using one vanishing point ($k = 1$).

Definition 1. Let A be a representation of a traditional house. The **house diagram** of A, denoted $\hat{A}$, is the projection of A onto a plane using k vanishing points. Formally we define the house diagram $\hat{A}$ using the function $D_k : A \mapsto \hat{A}$, where $D_k(A)$ represents the transformation of the house into a diagram with k vanishing points.

We then classified the many simple house forms into categories up to projective congruence. For example, based on the definition of projective transformations, the combination of a triangle and trapezoid is projective congruent with a rectangle-and-triangle configuration (Figure 3).

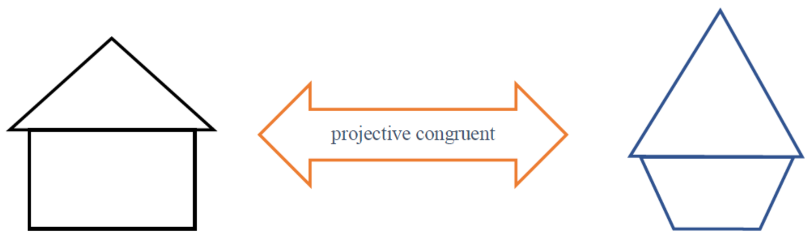


Figure 3: An example of projective congruence.

This identification up to congruence allowed us to develop building numbers for a variety of images of house types based on basic geometrical figures such as rectangles, triangles, the combination of rectangular triangle shapes, rectangular shapes, rectangular–triangular trapezoid shapes, rectangular trapezoid shapes, and shapes with curved edges. We list below the seven types of houses we will work with.

4.1.1. Simple House I (Cubic Simple House)

A simple house of type I has a house diagram as a cubic. It is the most basic form of a simple house. An example of this is a mud house in a local village in Pakistan; see Figure 4.

Figure 4: Example of a cubic simple house (Simple House I): a mud house in a local village in Pakistan.

Can you see that the diagram of Simple House I from one vanishing point shows it simply as a rectangle?

4.1.2. Simple House II (Triangular Simple House)

These traditional houses have a triangular prism shape. An example is the Uma Lengge presented in Figure 5. Uma Lengge are traditional houses in Bima District, West Nusa Tenggara, Sumbawa Island. These traditional houses, built on poles, are made of wood with thatched walls and are used for food storage and house members of the Mbojo tribe.

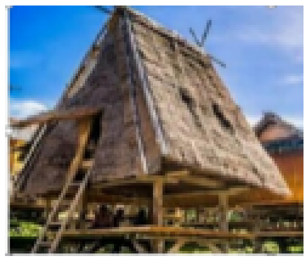


Figure 5: Example of triangular simple house (Simple House II): a house on Sumbawa Island.

Can you see that the diagram of Simple House II from one vanishing point shows it as a triangle?

4.1.3. Simple House III

Simple House III has a triangular–trapezoidal shape from the front view. Batak Toba traditional houses are an example of this traditional house; see Figure 6. The front part of a Batak Toba is triangles on the upper side and trapezoidal on the lower side.

Figure 6: Example of a triangular-trapezoidal house (Simple House III): Batak Toba traditional house.

Can you see what the diagram of Simple House III from one vanishing point would look like?

4.1.4. Simple House IV

The shape of Simple House IV combines a triangle and a rectangle. Some tribes in Indonesia use this model for a simple house. An example of this simple house can be observed in the Bone District, South Sulawesi Province; see for example Figure 7.

Figure 7: Example of a triangle-rectangle house (Simple House IV): house from the Bone District in Indonesia.

Can you see what the diagram of Simple House IV from one vanishing point would look like? Here is a hint: another example of Simple House IV was provided in Figure 2.

4.1.5. Simple House V

Two good examples for Simple House V are the Sao Mario House and Batak Toba House in Indonesia; see Figure 8.

Figure 8: Examples of a triangle-trapezoid-rectangle house (Simple House V): The Sao Mario House and Batak Toba House in Indonesia.

The house diagram of Simple House V from one vanishing point is defined by a combination of a triangle, a trapezoid, and a rectangle; can you see it?

4.1.6. Simple House VI

Two examples of this model are the Joglo house and Tongkonan House, located in Central Java, Indonesia; see Figure 9.

Figure 9: Examples of trapezoid-trapezoid-rectangle house (Simple House VI): Joglo and Tongkonan houses in Indonesia.

The geometric model of Simple House VI combines two trapezoids and one rectangle from one vanishing point. Can you see this? Can you visualize what happens with two vanishing points? How about three?

4.1.7. Simple House VII (Paraboloid Simple House)

Examples of Simple House VII are Hanoi houses in Papua Province, Indonesia; see Figure 10.

Figure 10: Example of parabola-like houses (Simple House VII): Hanoi houses in Indonesia.

The Simple House VII diagram displays a parabola-like roof no matter how many vanishing points are used. Why do you think this is?

4.2. Building Numbers and Projective Coordinates

We now assign a building number to each house diagram, in the following sense:

Definition 2. The **building numbers** of a house diagram $\hat{A}$ are in the form $\hat{n}_{\alpha_k}$, where α is the index of a building of the same type, k is the number of vanishing points, and n is a natural number.

In this article, n = 1, 2, 3, 4, 5, 6, and 7.

Next, the concept of projective coordinates must be introduced to identify a traditional house model. Then, this concept can be applied to traditional houses from Indonesia and other countries.

Definition 3. The 3-tuple $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ denotes the projective coordinate of a house diagram, giving building numbers from one vanishing point, two vanishing points, and three vanishing points, respectively.

Note: If *A and B are simple houses, A and B have the same projective coordinates if they are projective congruent.*

Traditional dwellings are classified using a coding system that uses building numbers and projective coordinates. In Table 1 we summarize simple house theory with respect to the seven simple house types we described earlier. Using these codification systems, we hope to eventually organize traditional dwellings from Indonesia and other nations in the future.

5. Methodology

5.1. Investigative site and participants

Data were taken from a collaboration course between UNM and Tunghai university as part of "MERDEKA BELAJAR KAMPUS MERDEKA ("Independent Campus, Freedom to Learn)" by the Ministry of education 2021. The observation of the students' engagement during the distance learning process. All students ware opening their cameras during the distance learning process. Students gave responses either in written form in google Jamboard or asked during the remote learning process. The subjects of this research were 36 students, 20 students from UNM Makassar (group I), and 16 from Tunghai University (Group II). Students from UNM Makassar are in international class programs in mathematics education, while Tunghai University's students are from the applied mathematics department.

5.2. Course Context

The course we taught was titled Ethnomathematics, and was specifically created to help students discover connections between mathematics and culture.

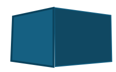
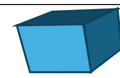
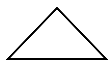
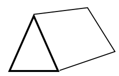
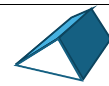
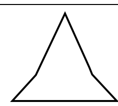
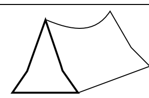
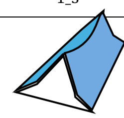
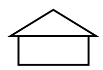
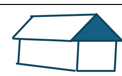
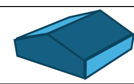
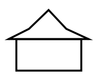
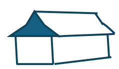
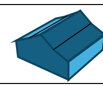
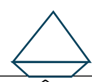
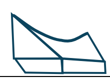
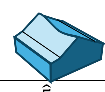
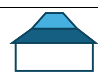
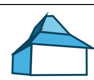
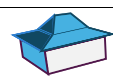
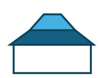
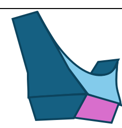
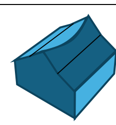
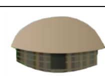
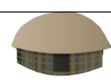
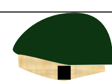
The type of Simple house.	***One vanishing point***	***Two vanishing point***	***Three vanishing point***	***Projective coordinates***
Simple House I (Cubic simple house)				
Building Numbers	$\hat{1}_{1\_1}$	$\hat{1}_{1\_2}$	$\hat{1}_{1\_3}$	$(\hat{1}_{1\_1}, \hat{1}_{1\_2}, \hat{1}_{1\_3})$.
Simple house II (Triangular simple house)				$(\hat{2}_{1\_1}, \hat{2}_{1\_2}, \hat{2}_{1\_3})$
Building Number	$\hat{2}_{1\_1}$	$\hat{2}_{1\_2}$	$\hat{2}_{1\_3}$	
Simple house III				$(\hat{3}_{1\_1}, \hat{3}_{1\_2}, \hat{3}_{1\_3})$
Building Number	$\hat{3}_{1\_1}$	$\hat{3}_{1\_2}$	$\hat{3}_{1\_3}$	
Simple house IV				$(\hat{4}_{1\_1}, \hat{4}_{1\_2}, \hat{4}_{1\_3})$
Building Number	$\hat{4}_{1\_1}$	$\hat{4}_{1\_2}$	$\hat{4}_{1\_3}$	
Simple house V				$(\hat{5}_{1\_1}, \hat{5}_{1\_2}, \hat{5}_{1\_3})$
Building Number	$\hat{5}_{1\_1}$	$\hat{5}_{1\_2}$	$\hat{5}_{1\_3}$	
Simple house V				$(\hat{5}_{1\_1}, \hat{5}_{2\_2}, \hat{5}_{2\_3})$
Building Number	$\hat{5}_{1\_1}$	$\hat{5}_{2\_2}$	$\hat{5}_{2\_3}$	
Simple house VI				$(\hat{6}_{1\_1}, \hat{6}_{1\_2}, \hat{6}_{1\_3})$
Building Number	$\hat{6}_{1\_1}$	$\hat{6}_{1\_2}$	$\hat{6}_{1\_3}$	
				$(\hat{6}_{1\_1}, \hat{6}_{2\_2}, \hat{6}_{2\_3})$
Building Number	$\hat{6}_{1\_1}$	$\hat{6}_{2\_2}$	$\hat{6}_{2\_3}$	
Simple house VII (Paraboloid simple house)				$(\hat{7}_{1\_1}, \hat{7}_{1\_1}, \hat{7}_{1\_1})$
Building Number	$\hat{7}_{1\_1}$	$\hat{7}_{1\_1}$	$\hat{7}_{1\_1}$	

Table 1: Summary of simple houses and their projective coordinates [5].

More specifically, the classes were designed to integrate ethnomathematics with projective geometry, focusing on the cultural and mathematical aspects of traditional house designs. The primary objective was to connect students' cultural backgrounds with mathematical concepts, fostering a deeper understanding and appreciation of both.

Simple House Theory was a central component of the course, introduced after students had a foundational understanding of projective geometry. As seen in Section 4, this theory involves the application of projective geometry principles to analyze and categorize traditional house designs. Students learned how to project these designs onto a plane using vanishing points, creating house diagrams that could be studied mathematically.

Then students were tasked with selecting a traditional house from their cultural background and applying Simple House Theory to create a house diagram. More information on the project, including the prompt and the rubric we used to grade it can be found in Appendix A.

5.3. The instruments

The instrument consists of survey , test dan projects. The survey included 27 closed-ended questions on 4 points Likert scale. The survey began with five questions about students' Engagement and Involvement in Distance Learning. The second part is related to clarity and encouragement and contains eight questions. The third part focused on learning outcomes which included 13 questions.

Ethnomathematics project rubrics and tests evaluate students' achievements.

5.4. Data collection

Procedures to collect data are:

1. Lecturers delivered their presentation by using Zoom, Meet, and teams, and students wrote their responses in google Jamboard and Pear Deck.
2. Students prepared their project.
3. Students presented their presentations through zoom.
4. Students gave responses in a survey about their group interaction.
5. The average time to fill out the survey is about 20 minutes.

6. Once students had completed the surveys and open-ended questions in google form, we analyzed the data.

5.5. Data analysis

Data were analyzed by quantitative and qualitative methods. Data numbers 1 to 22 were analyzed quantitatively using excel sheets. Data was collated into three categories: Students' engagement and involvement in distance learning, clarity and encouragement, and Learning outcome.

6. Results and Discussion

The level of student engagement in distance learning is measured by evaluating the level of interaction between students and the lecturer during the distance learning session. The quality of interaction also was measured.

During the distance learning, the researcher evaluated the implementation of Ethnomathematics in Projective geometry in three different sections which are (1) during lecturing by instructors, (2) during group discussion (3) During individual presentations

The interaction between students was measured by observation sheet by watching the video record during online learning. Two observers evaluated the learning activities. The result of the students' self-assessments used to maximize the students' interaction in online learning. The observation of the zoom recording shows that the student engages both in asynchronous learning and synchronous learning.

6.1. Student Engagement and Involvement in distance learning

Table 2 shows students' Engagement and Involvement in distance learning in projective geometry. Generally, although all the groups gave a positive response regarding their engagement and involvement in the process, the percentages of students in group I was significantly higher than those in group II.

In the table, two groups (Group I and Group II) were surveyed on various criteria related to their course experience. The criteria included "Enjoyment of Course," "Class Preparation," "Effort in Learning," "Challenge Level," and "Preparation for Discussion." The respondents were asked to choose from four options: 'Strongly Agree (SA)," "Agree (A)," "Disagree (D)," and "Strongly Disagree (SD)."

Criteria	Group I				Group II			
	SA	A	D	SD	SA	A	D	SD
Enjoyment of course	55	45	0	0	12.5	81.25	0	0
Class Preparation	55	45	0	0	6.25	62.5	25	0
Effort in Learning	30	45	25	0	6.25	62.5	12.5	0
Challenge Level	30	70	0	0	18.75	75	6.25	0
Preparation for Discussion	50	45	5	0	0	56.25	43.75	0

Table 2: Student Engagement and Involvement in distance learning. (Here SA = Strongly Agree, A = Agree, D = Disagree, SD = Strongly Disagree.)

From the data, it is evident that both groups had varied perceptions of their course experience. While Group II generally expressed higher agreement levels with aspects like enjoyment, class preparation, effort, and challenge level, they significantly differed in their views on preparation for discussion, with a substantial number expressing disagreement or strong disagreement.

6.2. The clarity and encouragement by the instructor in distance learning

Table 3 presents the data from the feedback from the two groups, Group I and Group II, regarding the clarity and encouragement provided by the instructor in distance learning. The entries are percentages.

Criteria	Group I				Group II			
	SA	A	D	SD	SA	A	D	SD
Explanation of Concepts	25	55	20	0	25	75	0	0
Helpful Feedback	65	35	0	0	25	75	0	0
Access to Instructor	60	35	0	0	18.75	56.25	25	0
Explanation of Concepts	25	55	20	0	25	75	0	0
Access to Instructor	60	35	0	0	18.75	56.25	25	0
Instructor Availability	50	45	0	0	25	75	0	0
Availability of Help	60	40	0	0	31.25	68.75	0	0
Instructor Care	60	40	0	0	37.5	56.25	0	0
Welcoming Environment	75	25	0	0	37.5	62.5	0	0
Respectful Treatment	60	40	0	0	25	75	0	0
Ability to Collaborate	40	60	0	0	18.75	62.5	18.75	0

Table 3: Student Engagement and Involvement in distance learning (percentages). (Here SA = Strongly Agree, A = Agree, D = Disagree, SD = Strongly Disagree.)

Overall, the data indicates that Group I generally showed higher levels of agreement with the instructor's performance across most criteria compared to Group II. However, Group II had areas that require attention and improvement, such as access to the instructor and the ability to collaborate, where a considerable percentage of respondents expressed disagreement. Enhancing the instructor's availability, providing clearer communication channels, and fostering collaborative activities can help create a more inclusive and interactive learning environment for Group II. It is essential to consider the limitations of the data, such as the small sample size of the two groups, which may limit the generalization of findings.

6.3. Student self assessment

Table 4 compares the self-assessment of the students about their achievement for 10 criteria which are developing critical thinking skills, evaluating arguments, solving problems, knowledge and confidence to communicate their understanding, recognizing and analyzing the relation between mathematics and culture as well as increasing the intercultural learning and awareness and utilization literature for their project.

Criteria	Group I				Group II			
	SA	A	D	SD	SA	A	D	SD
Development of Thinking Skills	75	25	0	0	25	68.75	0	0
Ability to Evaluate Arguments	55	45	0	0	6.25	87.5	6.25	0
Problem-solving Ability	40	60	0	0	18.75	81.25	0	0
Understanding of Knot Theory	45	55	0	0	12.5	87.5	0	0
Understanding of Simple House Theory	55	40	5	0	6.25	93.75	0	0
Relating Knot Theory to Culture	40	55	5	0	12.5	68.75	12.5	0
Relating Simple House Theory to Culture	60	35	5	0	12.5	81.25	6.25	0
Locating & Using Literature Information	40	55	5	0	12.5	75	6.25	0
Communicating Knot Theory	20	75	5	0	6.25	56.25	18.75	6.25
Communicating Simple House Theory	15	55	30	0	0	62.5	25	6.25
Analyzing Math-Knot Theory-Culture Relation	10	65	25	0	18.75	75	6.25	0
Analyzing Math-Simple House Theory-Culture Relation	50	50	0	0	6.25	87.5	6.25	0
Intercultural Knowledge & Global Citizenship	45	50	5	0	6.25	93.75	0	0

Table 4: Student Engagement and Involvement in distance learning. (Here SA = Strongly Agree, A = Agree, D = Disagree, SD = Strongly Disagree.)

Table 4 presents student self-assessment results, evaluating various criteria related to their learning experiences. The data reflects the percentage of students who expressed agreement or disagreement with different skills and knowledge areas.

Overall, the data portrays positive trends in thinking skills, understanding of concepts, and intercultural awareness. However, certain areas, such as effective communication and analysis of relation, require further attention. These results aid educators in adapting instructional approaches to cater to students' diverse needs and foster a conducive learning environment.

6.4. Students' achievements

Table 5 showcases the descriptive analysis of students' mathematics learning achievements through Ethnomathematics Projects.

Statistics	Result
Subjects	36
Ideal score	4
Highest score	3.68
Lowest score	3.36
Score range	0.32
Average score	3.56
Standard deviation of score	0.086

Table 5: Descriptive analysis of mathematics learning achievement (ethnomathematics Projects).

The data offers valuable insights into the overall performance of 36 subjects, demonstrating their scores' distribution and variance from the ideal score. The average score provides an overview of the students' collective achievement, while the standard deviation highlights the extent of variability in their individual scores. This data aids educators in evaluating the effectiveness of the Ethnomathematics Projects and may inform future instructional strategies to further enhance students' mathematics learning outcomes.

Table 6 presents the mathematics learning achievement level of students based on their participation in the Ethnomathematics Project. The table offers valuable insights into students' mathematics learning achievements, specifically concerning their participation in the Ethnomathematics Project.

Category	Frequency
Category	Frequency
Very Low	0
Low	0
High	20%
Very High	80%

Table 6: Mathematics learning achievement level.

Notably, no students are categorized as Very Low or Low achievers. The majority of students, 80%, demonstrate a Very High level of achievement, while 20% fall under the High category. These findings indicate the positive impact of the Ethnomathematics Project on students' mathematics learning outcomes, with a significant proportion of students achieving at a high level. Educators can leverage this information to enhance the effectiveness of the project and further support students' academic growth and success.

7. Discussion

7.1. Student Engagement and Involvement in Distance Learning

Student engagement is an important factor in education [6]. This research shows that the students can feel excited about and want to engage in distance learning. They can manage themselves to follow the learning process. Furthermore most students respond positively to the motivation survey.

The provided data presents a valuable insight into the perceptions and experiences of two groups (Group I and Group II) regarding their course. The data was collected by asking participants to indicate their level of agreement or disagreement on various criteria related to their course experience. Let us discuss the findings and implications of the data:

Enjoyment of Course

The data reveals that a higher percentage of respondents in Group II expressed agreement and strong agreement with enjoying the course compared to Group I. This suggests that the course content, teaching methods, or overall learning environment in Group II might have been more engaging and enjoyable for the participants. On the other hand, the lower percentage of respondents in Group I who agreed with enjoying the course may indicate that there were some aspects of the course experience that were less satisfying for this group.

Class Preparation

The responses on class preparation indicate a noticeable difference between the two groups. In Group II, a significantly lower proportion of respondents agreed with the statement, while a larger number remained neutral. This suggests that there might have been some issues with class preparation or organization in Group II, resulting in a lack of agreement among the participants. Conversely, in Group I, a higher percentage of respondents agreed with class preparation, implying that this aspect was handled better in their course.

Effort in Learning

The data on effort in learning shows a somewhat similar pattern between the two groups. Both groups had a considerable number of respondents who remained neutral on this criterion, suggesting that the perception of effort in learning might have been ambiguous or not strongly evident for many participants. However, a higher percentage of respondents in Group I disagreed with the effort in learning compared to Group II. This might indicate that participants in Group I felt they had to put in more effort to learn the course material.

Challenge Level

The findings related to the challenge level of the course reveal that a higher percentage of respondents in both groups strongly agreed with the appropriate challenge level. This indicates that the courses were perceived as appropriately challenging, fostering an environment conducive to learning. However, the difference between the two groups lies in the agreement level, with Group II having a lower percentage of respondents who agreed with the challenge level. This might imply that Group II participants found the course somewhat more challenging than Group I.

Preparation for Discussion

The most striking disparity in the data is observed in the preparation for discussion criterion. Group I had a significantly higher percentage of respondents who agreed with the preparation for discussion, while not a single respondent in Group II expressed agreement. Furthermore, a substantial number of respondents in Group II disagreed or strongly disagreed with the preparation for discussion. This points to a potential issue with the discussion preparation in Group II, which may have resulted in a less engaging and interactive learning experience for its participants.

The data provides valuable feedback for course organizers and educators to improve the learning experience for future participants. The higher levels of enjoyment, class preparation, and effort in learning perceived by participants in Group II can serve as positive examples to be emulated in future course designs. Conversely, the significant lack of agreement on preparation for discussion in Group II indicates a critical area that requires attention and improvement.

To address the disparity in the preparation for discussion, course facilitators can consider implementing more structured and well-organized discussion sessions. Providing clear guidelines and resources to participants can help foster a more engaging and participatory atmosphere. Additionally, seeking feedback from participants regarding their specific needs and preferences for discussion activities can contribute to a more tailored and enriching learning experience.

Moreover, the higher agreement levels on various criteria in Group II suggest that certain aspects of the course were more successful in this group. Identifying and replicating those successful elements in future courses could enhance participant satisfaction and learning outcomes.

7.2. The clarity and encouragement by the instructor in distance learning

Our results provide valuable insights into students' perceptions and feedback regarding the clarity and encouragement provided by instructors in distance learning. By recognizing and capitalizing on positive aspects and addressing areas for improvement, instructors can enhance the overall learning experience and better support students in their academic journey. It is essential to continually evaluate and refine instructional practices to ensure distance learning remains effective, engaging, and student-centered.

Table 3 presents significant findings regarding the clarity and encouragement provided by instructors in the context of distance learning, as perceived by students in both Group I and Group II. The table outlines nine distinct criteria that assess the instructor's role and effectiveness in distance learning. These criteria encompass the explanation of concepts, the provision of helpful feedback, accessibility of the instructor, their availability, the availability of help, the level of care demonstrated by the instructor, the creation of a welcoming environment, respectful treatment of students, and the fostering of a collaborative atmosphere.

For Group I, the agreement levels are remarkably high across all criteria, with the majority of students expressing agreement with the instructor's performance. This indicates that Group I students generally found the instructor to be clear in their explanations, accessible, caring, respectful, and successful in creating a collaborative and welcoming learning environment. Although four students disagreed with the clarity of concept explanations, this serves as a minor area for potential instructional improvement.

In contrast, Group II's overall agreement levels are slightly lower. Notably, lower agreement levels were observed for the 'Access to Instructor' and 'Ability to Collaborate' criteria. This suggests a perceived lack of instructor availability and a need for more efforts to foster collaborative learning in this group.

The distance learning environment, particularly when combined with the Ethnomathematics project, plays a crucial role in supporting students' learning experiences. One of the key factors contributing to the success of distance learning is the facilitation provided by the lecturers. Students thrive in an environment where lecturers demonstrate clarity in their explanations and encourage active engagement.

In this context, treating students with respect is essential, as it fosters a comfortable atmosphere where students feel valued and more likely to respond positively. A welcoming and inclusive learning environment emerges when students perceive mutual respect and care from their instructors. When lecturers show genuine care for students' progress, it motivates students to put in greater effort and dedication to their studies.

A significant aspect of distance learning is the provision of feedback. Students benefit immensely from timely feedback on their engagement, homework, and achievements. This feedback mechanism facilitates continuous improvement and helps students stay on track with their learning goals.

Furthermore, students must have access to facilitators whenever they encounter problems related to the course. Lecturers being readily available to help students when needed is vital to their success. Students greatly appreciate having access to instructors outside of scheduled class time, as it enables them to seek additional support and clarification.

Clear communication from the lecturer during the learning process is paramount for students to enhance their understanding. When instructors can effectively explain course material, it positively impacts students' performance and overall learning outcomes. For complex subjects like projective geometry, effective and illustrative explanations from the lecturers are crucial for students' comprehension.

For distance learning to be effective, instructors should possess certain key characteristics such as patience, availability, clarity, respect, and enthusiasm. These attributes contribute to creating a learning environment that is clear, engaging, and enjoyable for students. However, achieving such a model of distance learning requires substantial effort, preparation, and research-backed evidence demonstrating the lecturer's commitment to providing the best learning experience. This transformation shifts distance learning to a more student-centered approach, fostering greater autonomy and active participation.

7.3. Student achievement

Our results offer valuable insight into the positive impact of Ethnomathematics projects on students' critical thinking skills and understanding of concepts. However, there is room for improvement in certain areas, such as effective communication and the analysis of relations between different subjects and cultures. By acknowledging the positive aspects and addressing the challenges, educators can refine their instructional approaches and curricula to create a more enriching and empowering learning environment. Continuously promoting critical thinking, effective communication, and intercultural awareness will prepare students to be well-rounded and globally competent individuals.

The projects have proven effective in developing students' critical thinking skills. They encourage students to explore the connections between mathematics and their own culture, necessitating a deep understanding of both subjects. Through the projects, students connect with their traditions, buildings, and even create traditional tools to examine the relationship between mathematical concepts and their cultural practices. They then evaluate these connections and engage in problem-posing to assess their understanding.

One notable outcome of the Ethnomathematics projects is the improvement in students' ability to evaluate arguments based on the criteria provided to them. The projects allow students to formulate and solve problems based on their backgrounds, fostering confidence in communicating their newfound understanding. By recognizing and analyzing the relationship between mathematics and culture, students gain intercultural knowledge and develop a deeper awareness of their cultural identity [15].

The significance of assigning projects based on students' backgrounds becomes apparent as they immerse themselves deeply in their communities and surroundings to make their projects successful. Culture serves as a rich context that is highly relevant to mathematics and the real world, enhancing students' learning experiences and enabling them to achieve a higher level of learning.

While the research results reveal positive perceptions regarding students' critical thinking skills and understanding of concepts, certain weaknesses have also been identified. Specifically, effective communication is an essential skill for students to articulate their ideas clearly and engage in meaningful discussions. The lower levels of agreement in communicating Knot Theory and Simple House Theory suggest that students may benefit from additional support and guidance in effectively expressing their understanding of these topics.

The analysis of relations between Maths, Knot Theory, Simple House Theory, and Culture is a complex skill that requires critical thinking and interdisciplinary understanding. The mixed responses in this area may indicate that students would benefit from more opportunities to practice and explore these connections further.

On a positive note, the research results indicate that students are demonstrating intercultural knowledge and global citizenship awareness. This suggests that the educational curriculum successfully incorporates intercultural perspectives, contributing to students' awareness of and appreciation for diverse cultures.

To address the identified areas for improvement, educators can implement various strategies. Enhancing communication skills can be achieved through class discussions, presentations, and written assignments that encourage students to express their ideas effectively. Additionally, fostering collaborative learning environments can facilitate discussions and analysis of relations between different subjects and cultures.

Incorporating real-life examples and case studies can deepen students' understanding of the interconnections between mathematical concepts and their applications in various cultures. Encouraging research and inquiry-based learning can further empower students to explore and analyze complex relations independently.

8. Conclusion

Our results demonstrate the positive impact of student engagement and involvement in distance learning through Ethnomathematics projects. All groups responded positively, showing improvements in critical thinking and problem-solving skills. The data reveals differences between Group I and Group II perceptions, with Group II reporting higher enjoyment, class preparation, and effort in learning, while Group I had more agreement on preparation for discussion. Educators can use these insights to enhance course design and address areas for improvement to optimize participant satisfaction and learning outcomes.

Moreover, the research emphasizes the vital role of instructor clarity and encouragement in distance learning. Group I expressed higher agreement levels, indicating a positive perception of the instructor's performance, while Group II indicated areas for improvement in instructor availability and collaborative learning. Treating students with respect, providing timely feedback, and ensuring accessibility to instructors are crucial for success. Effective communication and clear explanations enhance understanding and learning outcomes, allowing a student-centered approach that fosters active participation and autonomy.

Additionally, the research highlights the positive impact of Ethnomathematics projects on students' critical thinking skills and understanding. These projects foster connections between mathematics and culture, improving argument evaluation and communication confidence. While intercultural knowledge and global citizenship awareness were evident, challenges include effective communication and analyzing relations between subjects and cultures. Implementing strategies like enhancing communication skills and fostering collaborative learning can address these challenges, preparing students to be well-rounded and globally competent individuals. Continuously evaluating and refining instructional practices will further enhance the effectiveness and satisfaction of distance learning environments.

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A. Student Project on Simple House Theory

After students were introduced to fundamental concepts of projective geometry and Simple House Theory, they were assigned individual projects to apply their learning. More specifically, students were tasked with selecting a traditional house from their cultural background and applying Simple House Theory to create a house diagram.

The prompt was as follows:

a. Choose a traditional house from your cultural background.

b. Apply the principles of projective geometry to create a house diagram with one or more vanishing points.

c. Analyze the geometric properties of the house diagram, including building numbers and projective coordinates.

d. Present your findings in a detailed report, explaining the mathematical and cultural significance of the house design.

The projects were evaluated based on the following criteria:

a. Understanding of Projective Geometry (30%): Demonstrates a clear understanding of projective geometry principles and their application to house diagrams.

b. Application of Simple House Theory (30%): Effectively applies Simple House Theory to analyze and categorize the chosen traditional house.

c. Cultural Relevance (20%): Connects the mathematical analysis to the cultural significance of the traditional house.

d. Presentation and Clarity (20%): Presents findings in a clear, well-organized, and visually appealing manner, including diagrams and explanations.