Global existence of wave maps on black hole background

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Abstract — During the past decade there has been a significant increase in interest in higher-dimensional black hole spacetimes, due to the recognition of their relevance to superstring/M-theory. Consider $D+1$-dimensional spherically symmetric spcetimes ($D \geq 3$). It is known already that one have black hole solutions to the Einstein equations under the stationary and some asymptotic conditions. From the viewpoint of the cosmic censorship, it is important to study whether such black hole spacetimes are stationary limit after dynamical evolution (i.e. naked singularities never appear by generic gravitational collapse) and/or the spacetimes are stable or not. It has been shown that $D+1$-dimensional static and spherical symmetric black hole spacetimes are stable against linear perturbation. As the next step, nonlinear perturbation should be considered. However, we have no mathematical tool to analyze nonlinear perturbation for curved spacetimes at the present time. Then, we will consider nonlinear scalar fields (as test fields) on the spacetimes. Our choice for the test fields are wave maps, which play an important role in classical general relativity. For example, dynamical evolution equations of the (four dimensional) Einstein equations for spacelike $U(1)$-symmetric spacetimes can be written as a wave map. Also, it is known that nonlinearity of wave maps is similar with one of the Einstein equations, that is "null form". This nonlinearity is a key to prove global existence theorems for the Einstein equations (for small initial data) and wave maps. Thus, wave maps are better choices as nonlinear test fields. Now, we show existence of global solutions of wave maps on static and spherically symmetric black hole spacetimes in higher-dimension.